Reflection and Transmission Characteristics of Coupled Wave through Micropolar Elastic Solid Interlayer in Micropolar Fluid

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Abstract

Based on micropolar fluid theory and micropolar solid elasticity theory, reflection and transmission characteristics of three kinds of micropolar elastic waves involving longitudinal displacement wave and two coupled waves, are studied when incident coupled wave propagates through micropolar elastic solid plate in micropolar fluid. Theoretical and numerical analytical results reveal that in general the amplitude ratios of various reflected and transmitted waves are functions of angle of incidence, frequency of the incident wave and of the material properties of the medium through which they travel. At normal incidence, the reflection and transmission of only coupled wave takes place and no other wave is found to reflect or transmit. At grazing incidence, no reflection or transmission phenomena take place and the same wave propagates along the interface. The rules of the reflection waves and refraction waves amplitudes varied with incident angle are also discussed.

Keywords

Micropolar Theory; Reflection; Transmission; Amplitude Ratios; Micropolar Elastic Wave

Introduction

Under continuum hypothesis of an elastic body, the classical theory of elasticity is based on linear stress–strain law (Hooke's law). In this theory, the transmission of load across a surface element of an elastic body is described by a force stress and the motion is characterized by translational degrees of freedom only. For materials possessing granular structure, it is found that the classical theory of elasticity is inadequate to represent the complete deformation. Certain discrepancies are observed between the results obtained experimentally and theoretically, particularly, in dynamical problems of waves and vibrations involving high frequencies. Cosserat and Cosserat (1909) was the first who gave importance to the microstructure of a granular body and incorporated a local rotation of points, in addition to the translation assumed in classical theory of elasticity. This theory is known as 'Cosserat theory' after their names. Mindlin (1964) presented a linear theory of a three-dimensional continuum having the properties of a crystal lattice, including the idea of unit cell. Later, Eringen incorporated micro-inertia and renamed the 'Cosserat elasticity' as the 'Micropolar elasticity'.The linear theory of micropolar elasticity developed by Eringen(1966) is basically an extension of the classical theory of elasticity.With rapid advance of the science and technology, the research in the space of the sandwich structure with the spread of the wave based on micropolar fluid theory has got more and more attention by scholars. The research of wave-absorbing materials is the most significant. Because the stealth and electromagnetic compatibility (EMC) technology become more and more important, the effection of electromagnetic wave absorption material is very outstanding. It becomes a modern military in the magic weapon of the electronic counter and a "secret weapon".

In recent decades, many problems of reflection and refraction of micropolar elastic waves at a plane interface have been studied by several researchers[1-10].

In this paper, based on micropolar fluid theory and micropolar solid elasticity theory, reflection and transmission characteristics of three kinds of micropolar elastic waves involving longitudinal displacement wave and two coupled waves are studied when incident coupled wave propagates through micropolar elastic solid plate in micropolar fluid. The change rules of the reflection waves and refraction waves amplitudes varied with incident angle are also discussed.

Equations of Motion and Constitutive Relations

In the absence of body force density and body couple

density, for micropolar fluid medium and micropolar solid medium, the equations of motion are given as Eqs.(1) and (2), respectively.

$$\left.\begin{aligned}&\left(c_{1\mathrm{f}}^2+c_{3\mathrm{f}}^2\right)\nabla\left(\nabla\cdot\dot{\mathbf{u}}^{\mathrm{f}}\right)-\left(c_{2\mathrm{f}}^2+c_{3\mathrm{f}}^2\right)\nabla\times\left(\nabla\times\dot{\mathbf{u}}^{\mathrm{f}}\right)\\&\qquad+c_{3\mathrm{f}}^2\nabla\times\dot{\boldsymbol{\Phi}}^{\mathrm{f}}=\ddot{\mathbf{u}}^{\mathrm{f}}\\&\left(c_{4\mathrm{f}}^2+c_{5\mathrm{f}}^2\right)\nabla\left(\nabla\cdot\dot{\boldsymbol{\Phi}}^{\mathrm{f}}\right)-c_{5\mathrm{f}}^2\nabla\times\left(\nabla\times\dot{\boldsymbol{\Phi}}^{\mathrm{f}}\right)\\&\qquad+c_{6\mathrm{f}}^2\left(\nabla\times\dot{\mathbf{u}}^{\mathrm{f}}-2\dot{\boldsymbol{\Phi}}^{\mathrm{f}}\right)=\ddot{\boldsymbol{\Phi}}^{\mathrm{f}}\end{aligned}\right\}\quad(1)$$

$$\left.\begin{aligned}&\left(c_{1\mathrm{s}}^2+c_{3\mathrm{s}}^2\right)\nabla\left(\nabla\cdot\mathbf{u}^{\mathrm{s}}\right)-\left(c_{2\mathrm{s}}^2+c_{3\mathrm{s}}^2\right)\nabla\times\left(\nabla\times\mathbf{u}^{\mathrm{s}}\right)\\&\qquad+c_{3\mathrm{s}}^2\nabla\times\boldsymbol{\Phi}^{\mathrm{s}}=\ddot{\mathbf{u}}^{\mathrm{s}}\\&\left(c_{4\mathrm{s}}^2+c_{5\mathrm{s}}^2\right)\nabla\left(\nabla\cdot\boldsymbol{\Phi}^{\mathrm{s}}\right)-c_{5\mathrm{s}}^2\nabla\times\left(\nabla\times\boldsymbol{\Phi}^{\mathrm{s}}\right)\\&\qquad+c_{6\mathrm{s}}^2\left(\nabla\times\mathbf{u}^{\mathrm{s}}-2\boldsymbol{\Phi}^{\mathrm{s}}\right)=\ddot{\boldsymbol{\Phi}}^{\mathrm{s}}\end{aligned}\right\}\quad(2)$$

where $c_{1\mathrm{r}}^2=\left(\lambda^{\mathrm{r}}+2\mu^{\mathrm{r}}\right)/\rho^{\mathrm{r}}$, $c_{2\mathrm{r}}^2=\mu^{\mathrm{r}}/\rho^{\mathrm{r}}$, $c_{3\mathrm{r}}^2=K^{\mathrm{r}}/\rho^{\mathrm{r}}$, $c_{4\mathrm{r}}^2=\left(\alpha^{\mathrm{r}}+\beta^{\mathrm{r}}\right)/\rho^{\mathrm{r}}j^{\mathrm{r}}$, $c_{5\mathrm{r}}^2=\gamma^{\mathrm{r}}/\rho^{\mathrm{r}}j^{\mathrm{r}}$, $c_{6\mathrm{r}}^2=c_{3\mathrm{r}}^2/j^{\mathrm{r}}$, ρ^{r} is the density of the medium, j^{r} is the micro-inertia and $\mathbf{u}^{\mathrm{r}}$ and $\boldsymbol{\Phi}^{\mathrm{r}}$ are, respectively, the displacement and microrotation vectors for the micropolar elastic half-spaces. Here, the quantity having superscript r corresponds to the fluid and solid medium when r = f and r = s, respectively. λ^{f} , μ^{f} and K^{f} are the fluid viscosity coefficients and α^{f} , β^{f} and γ^{f} are the fluid viscosity coefficients responsible for gyrational dissipation of the micropolar fluid, λ^{s} , μ^{s} are Lame's constant and K^{s} , α^{s} , β^{s} and γ^{s} are the micropolar elastic constants for the micropolar elastic solid.

For micropolar fluid medium and micropolar solid medium, the constitutive relations are given by Eqs.(3) and (4), respectively.

$$\left.\begin{aligned}\tau_{kl}^{\mathrm{f}}&=\lambda^{\mathrm{f}}\dot{u}_{r,r}^{\mathrm{f}}\delta_{kl}+\mu^{\mathrm{f}}\left(\dot{u}_{k,l}^{\mathrm{f}}+\dot{u}_{l,k}^{\mathrm{f}}\right)+K^{\mathrm{f}}\left(\dot{u}_{l,k}^{\mathrm{f}}-\varepsilon_{klp}\dot{\phi}_{p}^{\mathrm{f}}\right)\\m_{kl}^{\mathrm{f}}&=\alpha^{\mathrm{f}}\dot{\phi}_{r,r}^{\mathrm{f}}\delta_{kl}+\beta^{\mathrm{f}}\dot{\phi}_{k,l}^{\mathrm{f}}+\gamma^{\mathrm{f}}\dot{\phi}_{l,k}^{\mathrm{f}}\end{aligned}\right\}(3)$$

$$\left.\begin{aligned}\tau_{kl}^{\mathrm{s}}&=\lambda^{\mathrm{s}}u_{r,r}^{\mathrm{s}}\delta_{kl}+\mu^{\mathrm{s}}\left(u_{k,l}^{\mathrm{s}}+u_{l,k}^{\mathrm{s}}\right)+K^{\mathrm{s}}\left(u_{l,k}^{\mathrm{s}}-\varepsilon_{klp}\phi_{p}^{\mathrm{s}}\right)\\m_{kl}^{\mathrm{s}}&=\alpha^{\mathrm{s}}\phi_{r,r}^{\mathrm{s}}\delta_{kl}+\beta^{\mathrm{s}}\phi_{k,l}^{\mathrm{s}}+\gamma^{\mathrm{s}}\phi_{l,k}^{\mathrm{s}}\end{aligned}\right\}(4)$$

where τ_{kl}^{r} is the force stress tensor, m_{kl}^{r} is the couple stress tensor, the 'comma' in the subscript denotes the spatial derivative, δ_{kl} and ε_{klp} are Kronecker delta and the alternating tensors respectively. Other symbols have their usual meanings.

Using Helmholtz theorem, we can write

$$\begin{bmatrix}\mathbf{u}^{\mathrm{r}}\\\boldsymbol{\Phi}^{\mathrm{r}}\end{bmatrix}=\nabla\begin{bmatrix}A^{\mathrm{r}}\\C^{\mathrm{r}}\end{bmatrix}+\nabla\times\begin{bmatrix}\mathbf{B}^{\mathrm{r}}\\\mathbf{D}^{\mathrm{r}}\end{bmatrix},\quad\nabla\cdot\begin{bmatrix}\mathbf{B}^{\mathrm{r}}\\\mathbf{D}^{\mathrm{r}}\end{bmatrix}=0\qquad(5)$$

where A^{r} and C^{r} are the scalar potentials, while $\mathbf{B}^{\mathrm{r}}$ and $\mathbf{D}^{\mathrm{r}}$ are the vector potentials. Plugging Eqs.(5) into Eqs.(1), we can obtain

$$\Pi_1A^{\mathrm{f}}=0,\quad\Pi_2C^{\mathrm{f}}=0\qquad(6)$$

$$\left.\begin{aligned}\left(c_{2\mathrm{f}}^2+c_{3\mathrm{f}}^2\right)\nabla^2\dot{\mathbf{B}}^{\mathrm{f}}+c_{3\mathrm{f}}^2\nabla\times\dot{\mathbf{D}}^{\mathrm{f}}&=\ddot{\mathbf{B}}^{\mathrm{f}}\\c_{5\mathrm{f}}^2\nabla^2\dot{\mathbf{D}}^{\mathrm{f}}+c_{6\mathrm{f}}^2\nabla\times\dot{\mathbf{B}}^{\mathrm{f}}-2c_{6\mathrm{f}}^2\dot{\mathbf{D}}^{\mathrm{f}}&=\ddot{\mathbf{D}}^{\mathrm{f}}\end{aligned}\right\}\qquad(7)$$

where

$$\Pi_1=\left[\left(c_{1\mathrm{f}}^2+c_{3\mathrm{f}}^2\right)\nabla^2-\frac{\partial}{\partial t}\right]\frac{\partial}{\partial t}$$

and $\Pi_2=\left[\left(c_{4\mathrm{f}}^2+c_{5\mathrm{f}}^2\right)\nabla^2-2c_{6\mathrm{f}}^2-\frac{\partial}{\partial t}\right]\frac{\partial}{\partial t}$.

It can be seen that equations in Eqs.(6) are un-coupled in scalar potentials A^{r} and C^{r} while Eqs. (7) are coupled in vector potentials $\mathbf{B}^{\mathrm{r}}$ and $\mathbf{D}^{\mathrm{r}}$.

Plane Wave of Micropolar Fluid and Solid

Take the form of a plane wave propagating in the positive direction of a unit vector $\mathbf{n}$ as

$$\left\{A^{\mathrm{r}},C^{\mathrm{r}},\mathbf{B}^{\mathrm{r}},\mathbf{D}^{\mathrm{r}}\right\}=\left\{a^{\mathrm{r}},c^{\mathrm{r}},\mathbf{b}^{\mathrm{r}},\mathbf{d}^{\mathrm{r}}\right\}\exp\left\{ik\left(\mathbf{n}\cdot\mathbf{r}-Vt\right)\right\}\quad(8)$$

where a^{r} , c^{r} , $\mathbf{b}^{\mathrm{r}}$ and $\mathbf{d}^{\mathrm{r}}$ are constants, $\mathbf{r}\left(=x\vec{i}+y\vec{j}+z\vec{k}\right)$ is the position vector, V is the phase velocity in the direction of n,k $(=\omega/V)$ is the wave number, ω is the angular frequency.

Parfitt and Eringen (1969) have already shown that there exist four waves in an infinite micropolar elastic solid medium propagating with distinct phase velocities. The four phase velocities are given as follows.

(i) an independent longitudinal displacement wave propagating with phase velocity $V_{\mathrm{s}1}=c_{1\mathrm{s}}^2+c_{3\mathrm{s}}^2$

(ii) two sets of coupled waves, each consists of a transverse displacement wave and a transverse microrotational wave perpendicular to it, propagating with phase velocities $V_{\mathrm{s}2}$ and $V_{\mathrm{s}3}$ given by $V_{\mathrm{s}2,\mathrm{s}3}^2=\left(b\pm\sqrt{b^2-4ac}\right)/2a$, where

$$\omega_0^2=c_{6\mathrm{s}}^2,\ a=1-2\omega_0^2/\omega^2,$$

$$b=c_{2\mathrm{s}}^2+c_{3\mathrm{s}}^2+c_{5\mathrm{s}}^2-\left(2c_{2\mathrm{s}}^2+c_{3\mathrm{s}}^2\right)\omega_0^2/\omega^2,\ c=c_{5\mathrm{s}}^2\left(c_{2\mathrm{s}}^2+c_{3\mathrm{s}}^2\right)$$

(iii) an independent longitudinal microrotational wave propagating with phase velocity $V_{\mathrm{s}4}^2=\left(c_{4\mathrm{s}}^2+c_{5\mathrm{s}}^2\right)\left(1-2\omega_0^2/\omega^2\right)^{-1}$.

Similarly, Singh and Tomar have geiven that there exist four waves in an infinite micropolar fluid medium propagating with distinct phase velocities.

The four phase velocities are given by

$$V_{\mathrm{f1}}^2 = -i\omega\left(c_{\mathrm{1f}}^2 + c_{\mathrm{3f}}^2\right), V_{\mathrm{f2,f3}}^2 = \left(-b' \pm \sqrt{b'^2 - 4a'c'}\right)\Big/2a',$$

$$V_{\mathrm{f4}}^2 = -i\omega^2\left(c_{\mathrm{4f}}^2 + c_{\mathrm{5f}}^2\right)\left(\omega + 2ic_{\mathrm{6f}}^2\right)^{-1},$$

where

$$a' = \omega + 2ic_{\mathrm{6f}}^2,$$

$$b' = \omega\left[i\omega c_{\mathrm{5f}}^2 + i\left(c_{\mathrm{2f}}^2 + c_{\mathrm{3f}}^2\right)\left(\omega + 2ic_{\mathrm{6f}}^2\right) + c_{\mathrm{3f}}^2 c_{\mathrm{6f}}^2\right]$$

and $c' = -\omega^3 c_{\mathrm{5f}}^2\left(c_{\mathrm{2f}}^2 + c_{\mathrm{3f}}^2\right)$.

Reflection and Transmission of Coupled Wave

Introducing the Cartesian coordinates *x*, *y* and *z* such that *xy* plane($z=0$) is interface. Thickness of **d** micropolar solid placed between micropolar fluid , the z-axis is taken perpendicular to the interface ($z=0$) and pointing downward into the medium M_2 .We shall consider a two-dimensional problem in *xz* plane, so that the followings are the displacement and microrotational vectors in micropolar elastic solid and in micropolar fluid:

$$\mathbf{u}^{\mathrm{r}} = \left(u_1^{\mathrm{r}}(x,z), 0, u_3^{\mathrm{r}}(x,z)\right),\ \mathbf{\Phi}^{\mathrm{r}} = \left(0, \Phi_2^{\mathrm{r}}, 0\right)\ (\mathrm{r}=\mathrm{f,s}) \quad (9)$$

The plan couple wave incident to interface $z=0$ with phase velocity V_{f1} and incident angle θ_0 within micropolar fluid propagation and then entrance to micropolar solid medium M_2 and incident into interface $z=d$ go through medium M_2,then entrance to micropolar fluid medium M_1.

We take the following form of potentials:

Interface $z=0$:when $z \le 0$, in micropolar fluid medium M_1,

$$\left\{A^{\mathrm{f}}, B_2^{\mathrm{f}}, \phi_2^{\mathrm{f}}\right\}^T = \begin{Bmatrix} A_0\exp(\chi_0) + A_1'\exp(\chi_1') \\ \sum_{i=2}^{3} A_i'\exp(\chi_i'), \\ \sum_{i=2}^{3} \eta_i' A_i'\exp(\chi_i') \end{Bmatrix} \quad (10)$$

And when $0 \le z \le \mathrm{d}$, in micropolar solid medium M_2

$$\left\{A^{\mathrm{s}}, B_2^{\mathrm{s}}, \phi_2^{\mathrm{s}}\right\}^T = \begin{Bmatrix} A_1\exp(\chi_1) \\ \sum_{i=2}^{3} A_i\exp(\chi_i) \\ \sum_{i=2}^{3} \eta_i A_i\exp(\chi_i) \end{Bmatrix} \quad (11)$$

where A_0, A_1 and A_1' are the amplitude of the incident, reflected and refracted longitudinal displacement wave, respectively. A_2 , A_3 and A_2' , A_3' are the amplitude of the reflected and refracted couple wave .

$$\chi_0 = ik_1'\left(\sin\theta_0 x + \cos\theta_0 z\right) - i\omega_1' t,$$

$$\chi_i = ik_i\left(\sin\theta_i x + \cos\theta_i z\right) - i\omega_i t,$$

$$\chi_i' = ik_i'\left(\sin\theta_i' x - \cos\theta_i' z\right) - i\omega_i' t,\ (i=1,2,3).$$

When $d \le z$, in micropolar fluid medium M_1 , for longitudinal displacement wave:

$$\left.\begin{aligned} \left\{A^{\mathrm{s}}, B_2^{\mathrm{s}}, \phi_2^{\mathrm{s}}\right\} &= \begin{Bmatrix} A_1\exp(\chi_1) + B_1\exp(\chi_1^0), \\ \sum_{i=2}^{3} B_i\exp(\chi_i^0), \sum_{i=2}^{3} \eta_i B_i\exp(\chi_i^0) \end{Bmatrix} \\ \left\{A^{\mathrm{f}}, B_2^{\mathrm{f}}, \phi_2^{\mathrm{f}}\right\} &= \begin{Bmatrix} B_1'\exp(\chi_1''), \sum_{i=2}^{3} B_i'\exp(\chi_i''), \\ \sum_{i=2}^{3} \eta_i' B_i'\exp(\chi_i'') \end{Bmatrix} \end{aligned}\right\} \quad (12)$$

For couple wave I, $(j=2,3)$

$$\left.\begin{aligned} \left\{A^{\mathrm{s}}, B_2^{\mathrm{s}}, \phi_2^{\mathrm{s}}\right\}^T &= \begin{Bmatrix} B_4\exp(\chi_4^0) \\ A_2\exp(\chi_2) + \sum_{i=5}^{6} B_i\exp(\chi_i^0) \\ \sum_{i=5}^{6} \eta_j B_i\exp(\chi_i^0) \end{Bmatrix} \\ \left\{A^{\mathrm{f}}, B_2^{\mathrm{f}}, \phi_2^{\mathrm{f}}\right\}^T &= \begin{Bmatrix} B_4'\exp(\chi_4'') \\ \sum_{i=5}^{6} B_i'\exp(\chi_i'') \\ \sum_{i=5}^{6} \eta_j' B_i'\exp(\chi_i'') \end{Bmatrix} \end{aligned}\right\} \quad (13)$$

For couple wave II, $(j=2,3)$

$$\left.\begin{aligned} \left\{A^{\mathrm{s}}, B_2^{\mathrm{s}}, \phi_2^{\mathrm{s}}\right\}^T &= \begin{Bmatrix} B_7\exp(\chi_7^0) \\ \sum_{i=8}^{9} B_i\exp(\chi_i^0) \\ A_3\exp(\chi_3) + \sum_{i=8}^{9} \eta_j B_i\exp(\chi_i^0) \end{Bmatrix} \\ \left\{A^{\mathrm{f}}, B_2^{\mathrm{f}}, \phi_2^{\mathrm{f}}\right\}^T &= \begin{Bmatrix} B_7'\exp(\chi_7'') \\ \sum_{i=8}^{9} B_i'\exp(\chi_i'') \\ \sum_{i=8}^{9} \eta_j' B_i'\exp(\chi_i'') \end{Bmatrix} \end{aligned}\right\} \quad (14)$$

where B_i and B_i' are the amplitude of reflected and refracted longitudinal displacement wave or coupled wave, respectively.

$$\chi_i^0 = ik_i\left(\sin\varphi_i x - \cos\varphi_i z\right) - i\omega_i^0 t,$$

$\chi_i'' = ik_i'\left(\sin\varphi_i'' x + \cos\varphi_i'' z\right) - i\omega_i'' t$, $(i=1,2,\cdots,9)$. The coupling parameters $\eta_{2,3}$ and $\eta_{2,3}'$ are given by:

$$\eta_{2,3}=-c_{6\mathrm{s}}^{2}\left[V_{\mathrm{s2,s3}}^{2}-2\frac{c_{6\mathrm{s}}^{2}}{k_{2,3}^{2}}-c_{5\mathrm{s}}^{2}\right]^{-1},$$

$$\eta_{2,3}'=ic_{6\mathrm{f}}^{2}\left[\frac{V_{\mathrm{f2,f3}}}{k_{2,3}'}+2\frac{ic_{6\mathrm{f}}^{2}}{k_{2,3}'^{2}}+ic_{5\mathrm{f}}^{2}\right]^{-1}.$$

Boundary conditions to be satisfied at the interface $z=0$ and $z=d$, are the continuity of force stress, couple stress, displacement and microrotation. Mathematically, these boundary conditions can be written as:

$$\tau_{zz}^{\mathrm{s}}=\tau_{zz}^{\mathrm{f}},\tau_{zy}^{\mathrm{s}}=\tau_{zy}^{\mathrm{f}},m_{zy}^{\mathrm{s}}=m_{zy}^{\mathrm{f}},u_{1}^{\mathrm{s}}=u_{1}^{\mathrm{f}},u_{3}^{\mathrm{s}}=u_{3}^{\mathrm{f}},\phi_{2}^{\mathrm{s}}=\phi_{2}^{\mathrm{f}} \quad (15)$$

Employing the Snell's law given by:

$$\frac{\sin\theta_0}{V_{\mathrm{f1}}}=\frac{\sin\theta_i}{V_{\mathrm{si}}}=\frac{\sin\theta_i'}{V_{\mathrm{fi}}}=\frac{\sin\varphi_i}{V_{\mathrm{si}}}=\frac{\sin\varphi_{i+3}}{V_{\mathrm{si}}}=\frac{\sin\varphi_{i+6}}{V_{\mathrm{si}}}$$
$$=\frac{\sin\varphi_i'}{V_{\mathrm{fi}}}=\frac{\sin\varphi_{i+3}'}{V_{\mathrm{fi}}}=\frac{\sin\varphi_{i+6}'}{V_{\mathrm{fi}}},\quad (i=1,2,3)$$

and assuming that all frequencies are equal at the interface, 24 homogeneous equations can be obtained and can be written in a matrix form as:

$$\mathbf{PZ}=\mathbf{Q} \quad (16)$$

where

$$\mathbf{P}=\left[a_{ij}\right]_{24\times24},\ \mathbf{Z}=\left[z_k\right]^{T}\quad\left(k=1,2,\cdots,24\right),$$
$$\mathbf{Q}=\left[1\,1\,0\,1\,1\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\right]^{T}.$$

The entries of the matrix $\mathbf{P}$ and the elements of the matrix $\mathbf{Z}$ are given in Appendix.

Numerical Results and Discussions

The paper presents numerical example to explore the transmission characteristics of incident longitudinal displacement wave when throughing the micropolar fluids of micropolar elastic plate. For numerical computations, we take the values of the relevant parameters from Hsia(2006)[9].

For micropolar viscous fluid M_1:

$\lambda^{\mathrm{f}}=1.5\times10^{10}\mathrm{dyne\,s/cm^2}$,

$\mu^{\mathrm{f}}=0.3\times10^{10}\mathrm{dyne\,s/cm^2}$,

$K^{\mathrm{f}}=0.00223\times10^{10}\mathrm{dyne\,s/cm^2}$,

$\alpha^{\mathrm{f}}=0.00111\times10^{10}\mathrm{dyne\,s}$,

$\beta^{\mathrm{f}}=0.0022\times10^{10}\mathrm{dyne\,s}$,

$\gamma^{\mathrm{f}}=0.000222\times10^{10}\mathrm{dyne\,s}$,

$j^{\mathrm{f}}=0.0400\mathrm{cm^2}$,

$\rho^{\mathrm{f}}=0.8\,g/\mathrm{cm^3}$.

For micropolar elastic solid M_2:

$\lambda^{\mathrm{s}}=2.09730\times10^{10}\mathrm{dyne/cm^2}$,

$\mu^{\mathrm{s}}=0.91822\times10^{10}\mathrm{dyne/cm^2}$,

$K^{\mathrm{s}}=0.22965\times10^{10}\mathrm{dyne/cm^2}$,

$\alpha^{\mathrm{s}}=-0.0000291\times10^{10}\mathrm{dyne}$,

$\beta^{\mathrm{s}}=0.000045\times10^{10}\mathrm{dyne}$,

$\gamma^{\mathrm{s}}=0.0000423\times10^{10}\mathrm{dyne}$,

$j^{\mathrm{s}}=0.037\mathrm{cm^2}$,

$\rho^{\mathrm{s}}=0.0034\,g/\mathrm{cm^3}$.

And $\omega/\omega_0=100$ and thickness of the interlayer: d=0.01m. The system of equations given by Eqs.(16) are solved by Gauss elimination method. The values of the amplitude ratios are computed at different angles of incidence.

From FIG.1 to FIG.6,we get the curve of three kinds of wave reflection and transmission coefficient with the angle of incidence changed after coupled wave through micropolar elastic solid plate in micropolar fluid.

FIG.1 shows that the variation of the amplitude ratios of reflected longitudinal displacement waves with the incidence angle θ_0, when a plane coupled wave propagates from the micropolar fluid half-space with phase velocity V_{f2}. The amplitude ratio of longitudinal displacement wave has two peak value when incidence angle θ_0 is 27.7° or 68.6°, respectively. And it is zero when $\theta_0=45^{\circ}$.

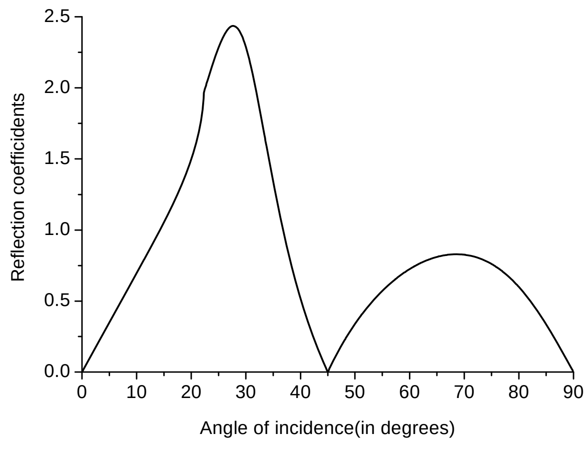

FIG. 1 REFLCTION COEFFICIENT OF LONGITUDINAL DISPLACEMENT WAVE

FIG.2 shows that the variation of the amplitude ratios of reflected coupled wave I with the incidence angle θ_0 .The amplitude ratio of coupled wave I has the minimal value when θ_0 is 19.1° . It decreases monotonically from the value 1.0 to the value 0.7784 at

$\theta_0 = 19.1^\circ$ and then it increases rapidly. The amplitude ratio of coupled wave II gets to the maximum value when θ_0 is about 23°. And then, the change of incidence angle has no effect on the amplitude ratio of coupled waves.

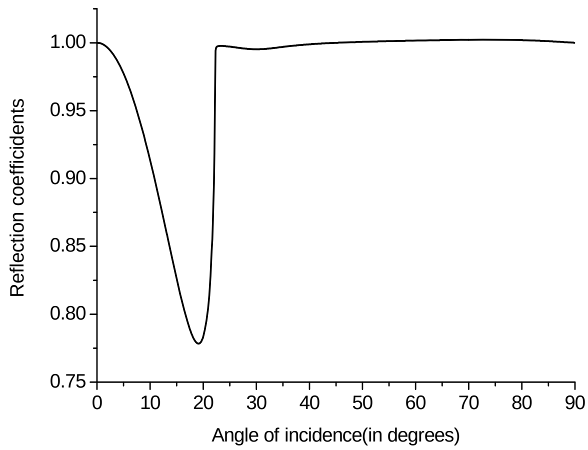

FIG. 2 REFLCTION COEFFICIENT OF COUPLE WAVE I

FIG.3 shows that the variation of the amplitude ratios of reflected coupled wave II with the incidence angle θ_0. The variation of the amplitude ratio of coupled wave II decreases slowly at first. It reaches the minimal value when θ_0 is about 20°. It suddenly rears up at about 23° in the range of 5°. It declines sharply to zero when θ_0 is about 45°. And then it gradually increases to the peak value at about 73°. At last, it decreases to zero gradually.

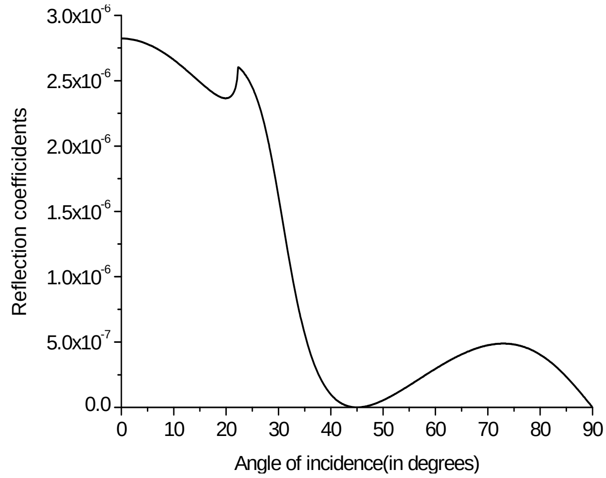

FIG. 3 REFLCTION COEFFICIENT OF COUPLE WAVE II

FIG.4 shows that the variation of the amplitude ratio of transmission longitudinal displacement wave with the incidence angle θ_0, when a plane coupled wave propagates from the micropolar fluid half-space with phase velocity V_{f2}. The amplitude ratio of longitudinal displacement wave has two peak value when incidence angle θ_0 is 28.3° or 73° ,respectively. The curve fluctuates at 23°. The amplitude ratio of of transmission longitudinal displacement wave is zero when θ_0 is about 45°.

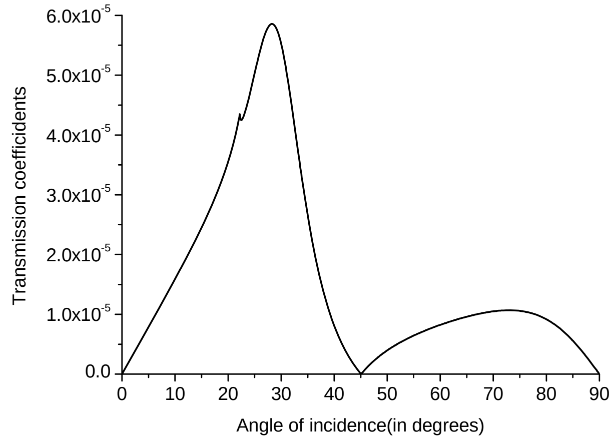

FIG. 4 TRANSMISSION COEFFICIENT OF LONGITUDINAL DISPLACEMENT WAVE

FIG.5 shows that the variation of the amplitude ratio of transmission couple wave I with the incidence angle θ_0. The variation of the amplitude ratio of coupled wave I changed slowly at first. Then it reaches to the peak value when θ_0 is about 30°. It declines sharply in the range of 10°. And then, the curve changes gently at about 67°. At last it decreases gradually to zero when θ_0 is 90°.

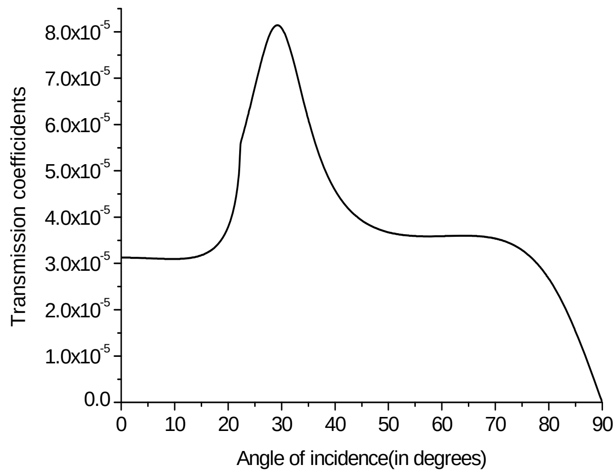

FIG. 5 TRANSMISSION COEFFICIENT OF COUPLE WAVE I

FIG.6 shows that the variation of the amplitude ratio of transmission couple wave II with the incidence angle θ_0. The shape of the curve likes the transmission couple wave I, but the amplitude ratio of couple wave II is small than that of couple wave I about four orders of magnitude.

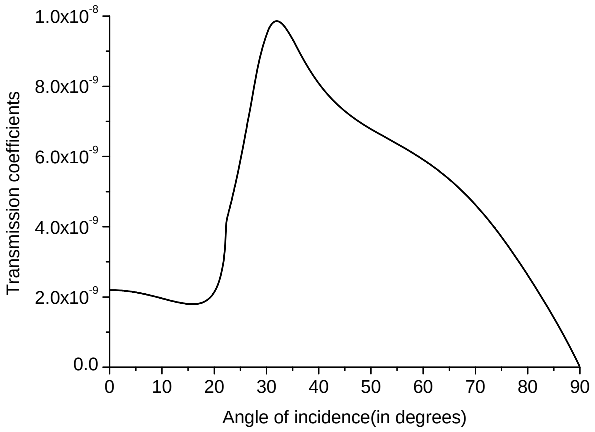

FIG. 6 TRANSMISSION COEFFICIENT OF COUPLE WAVE II

From FIG.1 and FIG.6, it can be seen that at normal incidence the reflection and transmission of only reflected couple wave I take place and no other wave is found to reflect or transmit. At grazing incidence no reflection or transmission phenomena take place and the same wave propagates along the interface.

Conclusions

Based on micropolar fluid theory and micropolar solid elasticity theory, the reflection and transmission phenomena of an coupled wave propagating through the micropolar elastic solid infinite plate in micropolar pluid was discussed. Theoretical and analytical results reveal that, in general, the amplitude ratios of various reflected and transmitted waves are functions of angle of incidence, of frequency of the incident wave and of the material properties of the medium through which they travel. Get three kinds of micropolar elastic wave's curves of reflection coefficient or transmission coefficient with the change of incident angle through the numerical results, and get the varying pattern of reflection and transmission coefficient: there is a maximum value in the curve of reflection coefficient or transmission coefficient of coupled wave,and there are two extremum in the curve of reflection coefficient or transmission coefficient of longitudinal displacement wave.

Based on the specific medium material parameters given by the paper ,we conclude that:

(1) At normal incidence, the reflection and transmission of only coupled waves take place and no longitudinal displacement wave is found to reflect or transmit. At grazing incidence, no reflection or transmission phenomena take place and the same wave propagates along the interface.

(2) There exist maximum values of reflection and transmission coefficient for coupled wave. There exist peak values of reflection and transmission coefficient for longitudinal displacement wave.

(3) There exist zero values of reflection and transmission coefficient for longitudinal displacement wave when incidence angle is 45°, at the same time, transmission coefficient presents wave phenomenon when incidence angle is 23°.

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Appendix

The component a_{ij} of the matrix $\mathbf{P}$ in non-dimensional form are given by

$a_{11} = L_1/L_0$, $a_{12} = -L_2/L_0$, $a_{12} = -L_3/L_0$, $a_{15} = L_4/L_0$, $a_{16} = L_5/L_0$, $a_{21} = M_1/M_0$, $a_{22} = M_4/M_0$, $a_{23} = M_5/M_0$, $a_{25} = -M_6/M_0$, $a_{26} = -M_7/M_0$, $a_{32} = -N_2$, $a_{33} = -N_3$, $a_{35} = -N_4$, $a_{36} = -N_5$, $a_{41} = P_1/P_0$, $a_{42} = -P_2/P_0$, $a_{43} = -P_3/P_0$, $a_{45} = P_4/P_0$, $a_{46} = P_5/P_0$, $a_{51} = Q_1/Q_0$, $a_{52} = Q_2/Q_0$, $a_{53} = Q_3/Q_0$, $a_{55} = -Q_4/Q_0$, $a_{56} = -Q_5/Q_0$, $a_{62} = \eta_2$, $a_{63} = \eta_3$, $a_{65} = -\eta_2'$, $a_{66} = -\eta_3'$, $a_{71} = L_1\xi_1^{-1}$, $a_{77} = L_1\xi_1$,$a_{78} = L_2\xi_2$,$a_{79} = L_3\xi_3$,$a_{710} = -L_0\xi_1'$,$a_{711} = -L_4\xi_2'$, $a_{712} = -L_5\xi_3'$, $a_{81} = M_1\xi_1^{-1}$, $a_{87} = -M_1\xi_1$, $a_{88} = M_4\xi_2$, $a_{89} = M_5\xi_3$, $a_{810} = -M_0\xi_1'$, $a_{811} = -M_6\xi_2'$, $a_{812} = -M_7\xi_3'$, $a_{98} = N_2\xi_2$, $a_{99} = N_3\xi_3$, $a_{911} = -N_4\xi_2'$, $a_{912} = -N_5\xi_3'$, $a_{101} = P_1\xi_1^{-1}$, $a_{107} = P_1\xi_1$, $a_{108} = P_2\xi_2$, $a_{109} = P_3\xi_3$, $a_{1010} = -P_0\xi_1'$, $a_{1011} = -P_4\xi_2'$, $a_{1012} = -P_5\xi_3'$, $a_{111} = Q_1\xi_1^{-1}$, $a_{117} = -Q_1\xi_1$, $a_{118} = Q_2\xi_2$, $a_{119} = Q_3\xi_3$, $a_{1110} = -Q_0\xi_1'$, $a_{1111} = -Q_4\xi_2'$, $a_{1112} = -Q_5\xi_3'$, $a_{128} = \eta_2\xi_2$, $a_{129} = \eta_3\xi_3$, $a_{1211} = -\eta_2'\xi_2'$, $a_{1212} = -\eta_3'\xi_3'$, $a_{1312} = -L_2\xi_2$, $a_{1313} = L_1\xi_1$, $a_{1314} = L_2\xi_2$, $a_{1315} = L_3\xi_3$, $a_{1316} = -L_0\xi_1'$, $a_{1317} = -L_4\xi_2'$, $a_{1318} = -L_5\xi_3'$, $a_{142} = M_2\xi_2^{-1}$, $a_{1413} = -M_1\xi_1$, $a_{1414} = M_4\xi_2$, $a_{1415} = M_5\xi_3$,$a_{1416} = -M_0\xi_1'$,$a_{1417} = -M_6\xi_2'$,$a_{1418} = -M_7\xi_3'$, $a_{1514} = N_2\xi_2$, $a_{1515} = N_3\xi_3$, $a_{1517} = -N_4\xi_2'$, $a_{1518} = -N_5\xi_3'$, $a_{162} = -P_2\xi_2^{-1}$, $a_{1613} = P_1\xi_1$, $a_{1614} = P_2\xi_2$, $a_{1615} = P_3\xi_3$, $a_{1616} = -P_0\xi_1'$, $a_{1617} = -P_4\xi_2'$, $a_{1618} = -P_5\xi_3'$, $a_{172} = Q_2\xi_2^{-1}$, $a_{173} = \eta_3$, $a_{1713} = -Q_1\xi_1$, $a_{1714} = Q_2\xi_2$, $a_{1715} = Q_3\xi_3$, $a_{1716} = -Q_0\xi_1'$, $a_{1717} = -Q_4\xi_2'$, $a_{1718} = -Q_5\xi_3'$, $a_{1814} = \eta_2\xi_2$, $a_{1815} = \eta_3\xi_3$, $a_{1817} = -\eta_2'\xi_2'$, $a_{1818} = -\eta_3'\xi_3'$, $a_{1919} = L_1\xi_1$, $a_{1920} = L_2\xi_2$, $a_{1921} = L_3\xi_3$, $a_{1922} = -L_0\xi_1'$, $a_{1923} = -L_4\xi_2'$, $a_{1924} = -L_5\xi_3'$, $a_{203} = M_3\xi_3^{-1}$, $a_{2019} = -M_1\xi_1$, $a_{2020} = M_4\xi_2$,

$a_{2021}=M_5\xi_3$, $a_{2022}=-M_0\xi_1'$, $a_{2023}=-M_6\xi_2'$, $a_{2024}=-M_7\xi_3'$, $a_{213}=N_1\xi_3^{-1}$, $a_{2120}=N_2\xi_2$, $a_{2121}=N_3\xi_3$, $a_{2123}=-N_4\xi_2'$, $a_{2124}=-N_5\xi_3'$, $a_{2219}=P_1\xi_1$, $a_{2220}=P_2\xi_2$, $a_{2221}=P_3\xi_3$, $a_{2222}=-P_0\xi_1'$, $a_{2223}=-P_4\xi_2'$, $a_{2224}=-P_5\xi_3'$, $a_{2319}=-Q_1\xi_1$, $a_{2320}=Q_2\xi_2$, $a_{2321}=Q_3\xi_3$, $a_{2322}=-Q_0\xi_1'$, $a_{2323}=-Q_4\xi_2'$, $a_{2324}=-Q_5\xi_3'$, $a_{243}=\xi_3^{-1}$, $a_{2420}=\eta_2\xi_2$, $a_{2421}=\eta_3\xi_3$, $a_{2423}=-\eta_2'\xi_2'$, $a_{2424}=-\eta_3'\xi_3'$, $-a_{14}=a_{24}=-a_{44}=a_{54}=1$, and other items are zero, where

$$L_0=i\omega k_1'^2\left[\lambda^{\mathrm{f}}+\left(2\mu^{\mathrm{f}}+K^{\mathrm{f}}\right)\cos^2\theta_0\right],$$

$$L_1=-k_1^2\left[\lambda^{\mathrm{s}}+\left(2\mu^{\mathrm{s}}+K^{\mathrm{s}}\right)\cos^2\theta_1\right],$$

$$L_{2,3}=k_{2,3}^2\left(2\mu^{\mathrm{s}}+K^{\mathrm{s}}\right)\sin\theta_{2,3}\cos\theta_{2,3},$$

$$L_{4,5}=i\omega k_{2,3}'^2\left(2\mu^{\mathrm{f}}+K^{\mathrm{f}}\right)\sin\theta_{2,3}'\cos\theta_{2,3}', M_3=-K^{\mathrm{s}},$$

$$M_0=i\omega k_1'^2\left(2\mu^{\mathrm{f}}+K^{\mathrm{f}}\right)\sin\theta_0\cos\theta_0,$$

$$M_1=-k_1^2\left(2\mu^{\mathrm{s}}+K^{\mathrm{s}}\right)\sin\theta_1\cos\theta_1,$$

$$M_2=k_2^2\left[\mu^{\mathrm{s}}\left(1-2\sin^2\theta_2\right)+K^{\mathrm{s}}\cos^2\theta_2\right],$$

$$M_{4,5}=k_{2,3}^2\left[\mu^{\mathrm{s}}\left(1-2\sin^2\theta_{2,3}\right)+K^{\mathrm{s}}\cos^2\theta_{2,3}\right]-K^{\mathrm{s}}\eta_{2,3},$$

$$M_{6,7}=-i\omega k_{2,3}'^2\left[\mu^{\mathrm{f}}\left(1-2\sin^2\theta_{2,3}'\right)+K^{\mathrm{f}}\cos^2\theta_2'\right]-K^{\mathrm{f}}\eta_{2,3}',$$

$$N_1=i\gamma^{\mathrm{s}}k_3\cos\theta_3, N_{2,3}=-i\gamma^{\mathrm{s}}\eta_{2,3}k_{2,3}\cos\theta_{2,3},$$

$$N_{4,5}=-\gamma^{\mathrm{f}}\eta_{2,3}'\omega k_{2,3}'\cos\theta_{2,3}', P_0=ik_1'\sin\theta_0, P_1=ik_1\sin\theta_1,$$

$$P_{2,3}=-ik_{2,3}\cos\theta_{2,3}, P_{4,5}=ik_{2,3}'\sin\theta_{2,3}', Q_0=ik_1'\cos\theta_0,$$

$$Q_{1,2,3}=ik_{1,2,3}\cos\theta_{1,2,3}, Q_{4,5}=ik_{2,3}'\sin\theta_{2,3}',$$

$$\xi_{1,2,3}=\exp\left(-ik_{1,2,3}\cos\theta_{1,2,3}d\right),$$

$$\xi_{1,2,3}'=\exp\left(ik_{1,2,3}'\cos\theta_{1,2,3}'d\right).$$

The vector Z: $Z_i=A_i/A_0$, $Z_{i+3}=A_i'/A_0$, $Z_{i+6}=B_i/A_0$, $Z_{i+9}=B_i'/A_0$, $Z_{i+12}=B_{i+3}/A_0$, $Z_{i+15}=B_{i+3}'/A_0$, $Z_{i+18}=B_{i+6}/A_0$, $Z_{i+21}=B_{i+6}'/A_0$. $(i=1,2,3)$.

where: when interface z=0, Z_1 、 Z_2 and Z_3 are the amplitude ratios for the refracted longitudinal displacement wave or coupled wave, respectively. Z_4, Z_5 and Z_6 are the amplitude ratios for the reflected longitudinal displacement wave or coupled wave, respectively;when interface z=d, Z_7, Z_8, Z_9, Z_{13}, Z_{14}, Z_{15}, Z_{19}, Z_{20} and Z_{21} are the amplitude ratios for the reflected longitudinal displacement wave or coupled wave, respectively, Z_{10}, Z_{11}, Z_{12}, Z_{16}, Z_{17}, Z_{18}, Z_{22}, Z_{23} and Z_{24} are the amplitude ratios for the refracted longitudinal displacement wave or coupled wave, respectively.

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